International Journal
of
Integrated Sciences & Technology (IJIST)
http://www.cuet.ac.bd/IJIST/index.html

International Journal of Integrated Sciences & Technology 3 (2018) 102-107

First Born Triple Differential Cross-Section for Ionization of H(3d) by Incident Electron at Different Energies

S. Banerjee* **and S. Dhar**

Department of Mathematics, Chittagong University of Engineering and Technology,
Chittagong 4349, Bangladesh

Abstract

Triple differential cross section of First Born approximation iscalculated for the ionization of metastable 3d-state hydrogen atoms by electron energy at 250 eV in the present work. A multiple scattering theory is applied in the present study.The present resultsare compared with the other related theoretical results for ionizations of hydrogen atoms from different metastable states. Obtainedresultshave anextensive scope for further study of such ionization process.

Keywords: *Electron, Ionization, Cross-Section, Scattering.*

1. Introduction

The analysis of multiple ionization of metastable atoms by electrons is of significantdemands in many experimentation field, more specially astrophysics, plasma physics and radiation physics as well as applied mathematics.The majorchallenge in the field of electron impact ionizationof atomis to develop a general theoretical framework, which willprovide an accurate ionization cross section for many atom over a practically relevant impact energy range. Due to its complexity, thefully quantum-mechanical treatment of electron impact ionization of atom is possible only for the simplest cases of hydrogen and helium. we used hydrogen atom targetin order to understand the ionization mechanism of atomicsystem by electron and positron impact.Hydrogen is the lightest and abundant substance in the universe. Ionization of hydrogen atoms by electrons is a good form for the perturbation theory. Ionization by fast particle was first treated quantum mechanically by Bethe [1]. Electron impact of single and double ionizationof hydrogen atom by electron,one of the most fundamentals of a process. Such electron-atom coincidence experiments called (e, 2e) experiments where the ejected electron is detected with the scattered electron. Over the last four decades, the study of triple differential cross-section (TDCS) in electron hydrogen atom ionization collision hasbecomegradually interesting. Using the (e, 2e) experiment, the study of TDCS were widely studied for ground state hydrogen atom both theoreti cally and experimentally whereas no such TDCS for the ionization of hydrogenic metastable states is yet available in the literature.

During the last five decades, Ionization of hydrogenatom byelectron have been considered to explorethe details of the ionization processboth in the groundstate [1-9] and metastable states [10-18] of atomic hydrogen.In this study, The First Born TDCS of the metastable 3d-state hydrogen atomsby electronsis evaluated at various kinematic conditions applying the multiple scattering theory of Das [2] and Das and Seal [8]. The present new study results will create a new dimensions on ionization of hydrogenic metastable states. Present results are compared with previous related theories [10, 17, 18].

**Corresponding author: sshmtbnrj53@gmail.com*

ISSN: 2411-9997

2. Theory

Ionization cross-section is the measure of the probability of the ionization process of atom by electron or molecule.

Electron-impact ionization cross section is estimated by taking the ratio of the number of ionization elements per unit time and per unit target to the incident electron flux. In this theory, we used a multiple scattering theory of Das and Seal [8].

Ionization of atomic hydrogen by electron in most elaborate form are presently available of following type

$$e^- + H(3d) \rightarrow H^+ + 2e^- \qquad (1)$$

Here 3d denotes the hydrogenic metastable state and has been attained in the coplanar geometry by examining TDCS measured in (e,2e) coincidence experiments.

The direct T-matrix element for ionization of hydrogen atoms by electrons, may be written as [8]

$$T_{fi} = \left\langle \Psi_f^{(-)}(\bar{r}_1,\bar{r}_2) \middle| V_i(\bar{r}_1,\bar{r}_2) \middle| \Phi_i(\bar{r}_1,\bar{r}_2) \right\rangle \qquad (2)$$

Here, $\overline{r_1}$ and $\overline{r_2}$ represent the coordinates of the atomic active electron and the incident electron, ($\overline{p_1}$, $\overline{p_2}$) and (E_1, E_2) represent the momenta and energies of the two electrons in the final state and ($\overline{p}_i, E_i$) are the momentum and the energy of the incident electron.

Where the perturbation potential $V_i(\bar{r}_1,\bar{r}_2)$ is given by

$$V_i(\bar{r}_1,\bar{r}_2) = \frac{1}{r_{12}} - \frac{Z}{r_2}$$

The nuclear charge of the hydrogen atom is Z=1, r_1 and r_2 are the distance of the two electrons from the nucleus and r_{12} is the distance between two electrons.

The initial channel unperturbed wave function is

$$\Phi_i(\bar{r}_1,\bar{r}_2) = \frac{e^{i\bar{p}_i.\bar{r}_2}}{(2\pi)^{3/2}}\phi_{3d}(\bar{r}_1)\,.$$

Where

$$\phi_{3d}(\bar{r}_1) = \frac{1}{81\sqrt{6\pi}}\left(r_1^{\,2}\right)\left(3\cos^2\theta - 1\right) e^{-r_1/3} \qquad (3)$$

Here $\lambda_1 = \frac{1}{3}$, $\phi_{3d}(\bar{r}_1)$ is the hydrogenic 3d-state wave function and $\Psi_f^{(-)}(\bar{r}_1,\bar{r}_2)$ is approximate wave function is given by [8]

$$\Psi_f^{(-)}(\bar{r}_1,\bar{r}_2) = N(\bar{p}_1,\bar{p}_2)\left[\phi_{\bar{p}_1}^{(-)}(\bar{r}_1)e^{i\bar{p}_2.\bar{r}_2} + \phi_{\bar{p}_2}^{(-)}(\bar{r}_2)e^{i\bar{p}_1.\bar{r}_1} + \phi_{\bar{p}}^{(-)}(\bar{r})e^{i\bar{P}.\bar{R}} - 2e^{i\bar{p}_1.\bar{r}_1 + i\bar{p}_2.\bar{r}_2}\right]$$
$$/(2\pi)^3 \qquad (4)$$

where

$$\bar{r} = \frac{\bar{r}_2 - \bar{r}_1}{2},\ \bar{R} = \frac{\bar{r}_1 + \bar{r}_2}{2},$$

$$\bar{p} = (\bar{p}_2 - \bar{p}_1),\ \bar{P} = \bar{p}_2 + \bar{p}_1,$$

The triple differential cross section is denoted by the symbol $\frac{d^3\sigma}{d\Omega_1 d\Omega_2 dE_1}$.

and the Coulomb wave function $\phi_q^{(-)}(\bar{r})$ is given by

$$\phi_q^{(-)}(\bar{r}) = e^{\pi\alpha/2}\Gamma(1+i\alpha)e^{i\bar{q}.\bar{r}}F_1(-i\alpha,1,-i[qr+\bar{q}.\bar{r}])$$

with

$$\alpha_1 = \frac{1}{p_1} \text{ for } \bar{q} = \overline{p_1}$$

$$\alpha_2 = \frac{1}{p_2} \text{ for } \bar{q} = \overline{p_2}$$

and

$$\alpha = -\frac{1}{p} \text{ for } \bar{q} = \bar{p}$$

Now equation (2) becomes

$$T_{fi} = T_B + T_{B'} + T_i - 2T_{PB} \qquad (5)$$

where

$$T_{\mathbf{B}} = \left\langle \phi_{p_1}^{(-)}(\bar{r}_1)\, e^{i\bar{p}_2.\bar{r}_2} \left| V_i \right| \Phi_i(\bar{r}_1, \bar{r}_2) \right\rangle \qquad (6)$$

$$T_{B'} = \left\langle \phi_{p_2}^{(-)}(\bar{r}_2)\, e^{i\bar{p}_1.\bar{r}_1} \left| V_i \right| \Phi_i(\bar{r}_1, \bar{r}_2) \right\rangle \qquad (7)$$

$$T_i = \left\langle \phi_{p}^{(-)}(\bar{r})\, e^{i\bar{P}.\bar{R}} \left| V_i \right| \Phi_i(\bar{r}_1, \bar{r}_2) \right\rangle \qquad (8)$$

$$T_{PB} = \left\langle e^{i\bar{p}_1.\bar{r}_1 + i\bar{p}_2.\bar{r}_2} \left| V_i \right| \Phi_i(\bar{r}_1, \bar{r}_2) \right\rangle \qquad (9)$$

Here equation (6) is called First Born term and it may be written as

$$T_{\mathbf{B}} = \frac{1}{324\sqrt{3}\pi^2} \left\langle \phi_{p_1}^{(-)}(\bar{r}_1) e^{i\bar{p}_2.\bar{r}_2} \left| \frac{1}{r_{12}} - \frac{1}{r_2} \right| e^{i\bar{p}_i.\bar{r}_2} \left(r_1^{\ 2}\right)\left(3\cos^2\theta - 1\right) e^{-r_1\lambda_1} \right\rangle$$

$$= \frac{1}{324\sqrt{3}\pi^2} \int \phi_{p_1}^{(-)*}(\bar{r}_1) e^{-i\bar{p}_2.\bar{r}_2} \left(\frac{1}{r_{12}} - \frac{1}{r_2} \right) e^{i\bar{p}_i.\bar{r}_2} \left(r_1^{\ 2}\right)\left(3\cos^2\theta - 1\right) e^{-\lambda_1 r_1} d^3r_1 d^3r_2$$

$$T_B = \frac{1}{324\sqrt{3}\pi^2} \int \phi_{p_1}^{(-)*}(\bar{r}_1) e^{-i\bar{p}_2.\bar{r}_2} \frac{1}{r_{12}} e^{i\bar{p}_i.\bar{r}_2} r_1^{\ 2}\left(3\cos^2\theta - 1\right) e^{-\lambda_1 r_1} d^3r_1 d^3r_2$$

$$- \frac{1}{324\sqrt{3}\pi^2} \int \phi_{p_1}^{(-)*}(\bar{r}_1) e^{-i\bar{p}_2.\bar{r}_2} \frac{1}{r_2} e^{i\bar{p}_i.\bar{r}_2} r_1^{\ 2}\left(3\cos^2\theta - 1\right) e^{-\lambda_1 r_1} d^3r_1 d^3r_2$$

$$T_{\mathbf{B}} = \mathrm{tb1} + \mathrm{tb2} \qquad (10)$$

where

$$\mathrm{tb1} = \frac{1}{324\sqrt{3}\pi^2} \int \phi_{p_1}^{(-)*}(\bar{r}_1) e^{-i\bar{p}_2.\bar{r}_2} \frac{1}{r_{12}} e^{i\bar{p}_i.\bar{r}_2} r_1^{\ 2}\left(3\cos^2\theta - 1\right) e^{-\lambda_1 r_1} d^3r_1 d^3r_2$$

$$\mathrm{tb2} = -\frac{1}{324\sqrt{3}\pi^2} \int \phi_{p_1}^{(-)*}(\bar{r}_1) e^{-i\bar{p}_2.\bar{r}_2} \frac{1}{r_2} e^{i\bar{p}_i.\bar{r}_2} r_1^{\ 2}\left(3\cos^2\theta - 1\right) e^{-\lambda_1 r_1} d^3r_1 d^3r_2$$

The terms of equation (10) is calculated for First Born approximation.

After analytical calculation by using the Lewis integral [19], The First Born term $T_{\mathbf{B}}$ of equation (6) is evaluated numerically using the computer language MATLAB. Finally the triple differential cross-sections for T-Matrix element is given by

$$\frac{d^3\sigma}{d\Omega_1 d\Omega_2 dE_1} = \frac{p_1 p_2}{p_i} \left| T_{fi} \right|^2 \qquad (11)$$

3. Results and Discussions

The present results of triple differential cross section are presented from the Fig. 1 to Fig. 6 The incident energy is taken $E_i = 250$ eV and ejected energy is $E_1 = 5$ eV where the ejected angle θ_1 varies and the scattered angles θ_2 are fixed. The scattering angles are $\theta_2 = 5^o$ (Fig. 1.) , 7^o (Fig. 2), 9^o (Fig. 3) 11^o (Fig. 4), 15^o (Fig. 5) and 20^o (Fig. 6) where the ejected angle θ_1 varies from 0^o to 360^o . In all figures, we consider, $\phi = 0^o$ as recoil region while $\phi = 180^o$ as binary region.

The results of hydrogenic metastable 2S, 3S and 3P states are presented here for comparison with our present result. It is seen that, the present First Born result and the previous measurement of 3P state hydrogen atom First Born results are nearly identical for all scattering angles.

Here in the Fig. 1 and Fig. 2, the present and 3P metastable state results approach with one deep lobed at ejected angle about 180^o and 36^o respectively. On the other hand, present First Born results and the metastable 2S state results demonstrate dissimilar peak in the recoil region and complete opposite peak in the binary region.

Let us consider the case of Fig. 3, the present TDCS result shows a very sharp peak in the recoil region whereas the peak of 2S-state results is very large magnitude in the binary region. The metastable 3d-state and 3P-state results exhibits a short lobed at ejected angle about 108^o.

In Fig. 4, in the case of recoil and binary region, the peak values of 3d-state results are distinct with 3S-state results.

In Fig. 5, in binary region , the peak height of 3d-state and 3P-state results is increased in higher ejected angle about 324^o.

Lastly, in Fig. 6, Present TDCS and 3P-state results display higher magnitude in both recoil and binary regions than the corresponding compared result of 2S metastable state.

Finally, Metastable 3d-state is an excited state of an atom or other system with a longer lifetime than the other excited states. However, it has a shorter lifetime than the stable ground state. The peak structure of the present results show good qualitative agreement with compared result in the recoil region but show somewhat disagreement in the binary region. This may be happened due to the change of the hydrogenic metastable states by electrons. It is remarked that, the peak structure for both in recoil and binary region, the First Born results are very close to the 3P-metasble state with different magnitude for all scattering angles. But in the binary region, the opposite peak patterns of 2S-mathastable state [10] are very sharp than the corresponding 3P-state [18] and 3d-state results. Here a table (please see **Table1**.) of comparison results for ionization of hydrogenic 2S-state, 3S-state and 3P-state atoms by electron is presented.

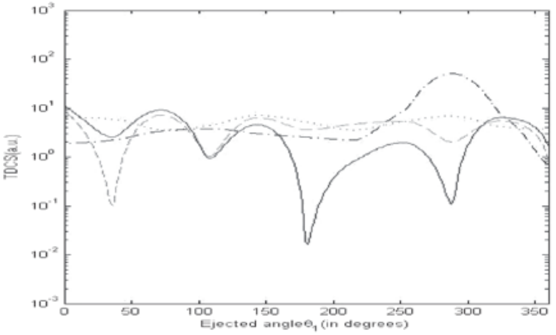

Fig. 1: Triple-differential cross sections (TDCS) versus ejected electron angle θ_1 for electron impact energy $E_i = 250$ eV, ejected electron energy $E_1 = 5$ eV and for scattering angle $\theta_2 = 5^o$. Theory: Continuous curve : Present First born result, dash curve : 3P-state First born result [18],dotted curve : 3S-state First born result [17] and dash dotted curve : 2S-state First born result [10].

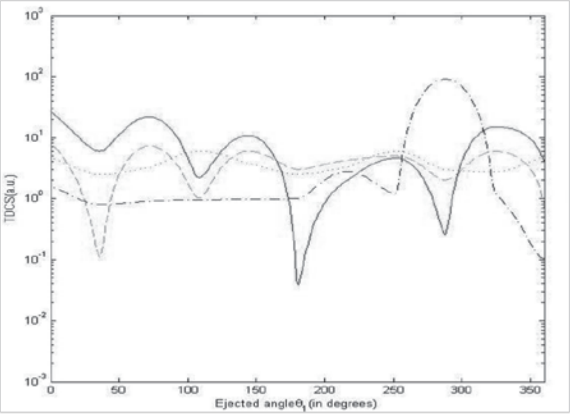

Fig. 2: TDCS versus ejected electron angle θ_1 for electron impact energy $E_i = 250$ eV ejected electron energy $E_1 = 5$ eV and for scattering angle $\theta_2 = 7^o$. Theory: Continuous curve : Present First born result, dash curve : 3P-state First born result [18], dotted curve : 3S-state First born result [17] and dash dotted curve : 2S-state First born result [10]

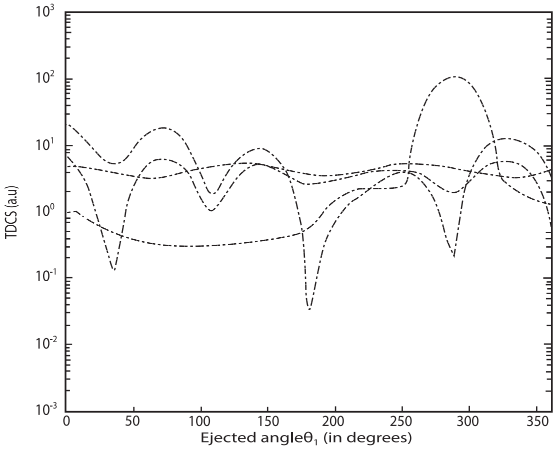

Fig. 3: TDCS versus ejected electron angle θ_1 for electron impact energy $E_i = 250$ eV ejected electron energy $E_1 = 5$ eV and for scattering angle $\theta_2 = 9^o$. Theory: Continuous curve : Present First born result, dash curve : 3P state First born result [18],dotted curve : 3S-state First born result [17] and dash dotted curve : 2S-state First born result [10].

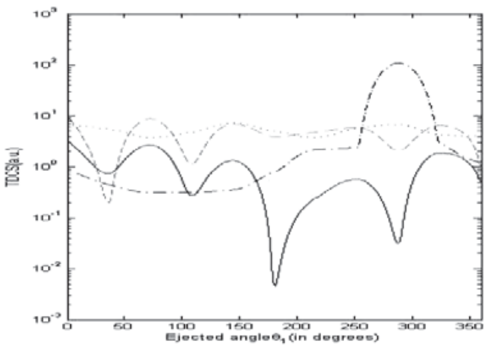

Fig. 4: TDCS versus ejected electron angle θ_1 for electron impact energy $E_i = 250\,\text{eV}$, ejected electron energy $E_1 = 5\,\text{eV}$ and for scattering angle $\theta_2 = 11^o$. Theory: Continuous curve : Present First born result, dash curve: 3P-state First born result [18], dotted curve: 3S-state First born result [17] and dash dotted curve: 2S-state First bornresult [10].

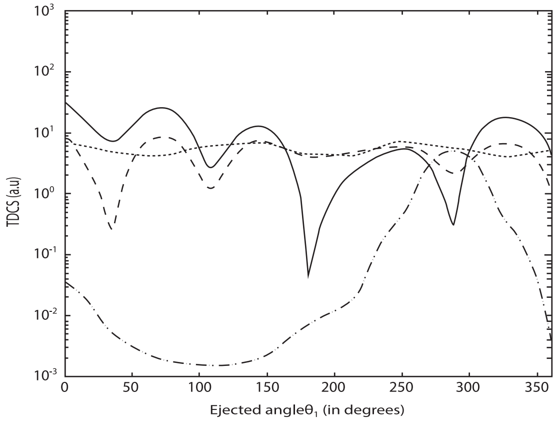

Fig. 5: Triple-differential cross sections (TDCS) versus ejected electron angle θ_1 for electron impact energy $E_i = 250\,\text{eV}$, ejected electron energy $E_1 = 5\,\text{eV}$ and for scattering angle $\theta_2 = 15^o$. Theory : Continuous curve : Present First born result, dash curve : 3P-state First born result [18], dotted curve: 3S-state First born result [17] and dash dotted curve: 2S-state First born result [10].

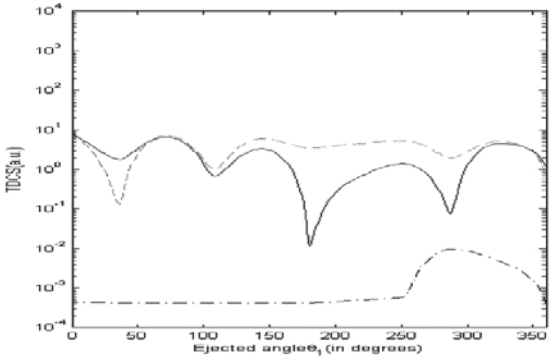

Fig. 6: TDCS versus ejected electron angle θ_1 for electron impact energy $E_i = 250\,\text{eV}$, ejected electron energy $E_1 = 5\,\text{eV}$ and for scattering angle $\theta_2 = 20^o$. Theory: Continuous curve : Present First born result, dash curve : 3P-state First born result [18] and dash dotted curve : 2S-state First born result [10].

Table-1: Triple differential cross section (TDCS) for ionization of atomic hydrogen atoms by electron impact at metastable 3d-state are obtained by using equation (11). The incident energy is 250eV, the scattering angle is $\theta_2 = 11^0$ and the ejected electron energy is $E_1 = 5eV$. In the given table we present 3d-state First born results and compared 3 P-state, 3S-state & 2S-state First born results.

Ejected angle (θ_1)	2S	3S	3P	3d
0	0.8501	6.7915	9.5031	3.2694
36	0.4513	5.5123	0.2001	0.7393
72	0.3101	3.7612	8.7523	2.6675
108	0.3150	4.7330	1.1532	0.2722
144	0.3550	7.2575	7.0012	1.3320
180	0.7910	5.0013	3.9324	0.0045
216	2.1693	3.7503	4.7590	0.2438
252	2.4011	5.7301	5.9831	0.5707
288	109.00	6.8103	2.1139	0.0314
324	2.9823	4.2001	6.5371	1.8645
360	1.2711	4.000	0.8973	0.4702

4. Conclusions

In this work, theTDCS for ionization of atomic hydrogenby electron in metastable 3d-state is computed usingthe multiple scattering theory of Das and Seal[8]. The present results showa very attractive binary peak features. The present new computational result gives significant contribution in the field of metastable state ionization problem. Itwill be addeda new dimension to the study of the ionization problem. Calculation for other kinematic condition or other atomic spices will also be interesting.

Acknowledgements

The computational works have been performed in the Simulation Lab of the Department of Mathematics, Chittagong University of Engineering and Technology, Chittagong-4349, Bangladesh.

References

[1] H. Bethe, Annalen der Physik, 397, 325 0(1930)
[2] J. N. Das, Physical ReviewA, 42, 1376 (1990)
[3] J. N. Das and S. Seal, Pramana, 40, 253 (1993)
[4] J. N. Das and Chakraborty, Phys. Rev. A32,176(1985)
[5] C. J. Joachain and B. Piraux, Comments at. Mol. Phys.,17261 (1986)
[6] H. Ehrhardt, K. Jung, G. Knoth, and P. Schlemmer, Zeitschriftfür Physik D Atoms, Molecules and Clusters, 1, 3 (1986)
[7] F. W. Byron, C. J. Joachen and B. Piraux, Journal of Physics B: Atomic, Molecular and Optical Physic.13, L673 (1980)
[8] J. N. Das and S Seal, Physical ReviewA, 47, 2978 (1993)
[9] M Brauner, J. S. Briggs and H. Klar. Journal of Physics B: Atomic, Molecular and OpticalPhysics.22, 2265 (1989)
[10] S. Dhar, Australian J. Phy., 49, 937 (1996)
[11] J. N. Das and S. Dhar, Pramana, J. Phys, 47, 263 (1999)
[12] S. Vučič, R. M. Potvliegeand C. J. Joachain, Physical Review A35, 1446 (1987)
[13] Y. Y. Qi, L. N. Ning, J. G. Wang and Y. Z. Qu, Phys. Plasmas 20,123301(2013)
[14] S. Dhar and N. Nahar, Results in Phys., 4, 170 (2014)
[15] S. Dhar and N. Nahar., Open Journal of Microphysics, 4, 46 (2014)
[16] S. Dhar and N. Nahar.American Journal of Modern Phys., 4, 132 (2015)
[17] S. Dhar, T. Noor and F.S. Chowdhury, Ame. J. Mod. Phys., 4, 261 (2015)
[18] S. Dhar, S. Akter and N. Nahar, Open Journal of Microphysics, 6, 15 (2016)
[19] R. R. Lewis, Phys. Revi. 102, 537. (1956)